

PRIFYSGOL CYMRU : UNIVERSITY OF WALES

**DEGREE EXAMINATIONS 2012
ARHOLIADAU GRADD 2012**

**Swansea Metropolitan University
Prifysgol Fetropolitan Abertawe**

Faculty of Applied Design and Engineering

NAME OF COURSE/PROGRAMME: *BEng Motorcycle Engineering*

YEAR or LEVEL (state which): *Year II*

NAME OF MODULE/UNIT: *Motorcycle Dynamics*

NAME OF EXAM PAPER:
(if different to module name)

DATE OF EXAMINATION:

EXAMINATION TIMES:

Start Time:

Finish Time:

Time Allowed:

INSTRUCTIONS TO CANDIDATES:

For all questions, take $\rho_{air} = 1.167 \text{ kg m}^{-3}$

An equation sheet is provided with this exam paper.

Answer 3 from 4 Questions

FACULTY REF:

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Question 1 – (Rectilinear Dynamics)

- a. Derive a set of equations that can be used to determine the wheel loads of a motorcycle in both static and dynamic situations.

(8 marks)

Consider a motorcycle with the following properties:

Mass with rider, $m = 181\text{kg}$

Reduced Mass, $m^* = 235\text{kg}$

Frontal Area, $A = 0.6\text{ m}^2$

Drag Coefficient, $C_D = 0.6$

Wheelbase, $p = 1.3\text{m}$

50/50 weight distribution

Front vertical stiffness $k_f = 18.5\text{ N/mm}$

Rake angle = 22.3°

- b. What is the *total* driving force required to maintain the motorcycle at a constant velocity of 155 kmh^{-1} along a horizontal flat road?

(3 marks)

- c. If the same motorcycle is required to climb an incline of 28° , what power would the motorcycle be required to produce?

(2 marks)

- d. If the motorcycle has a maximum power output of 68KW , what would you expect its aerodynamically limited top speed to be?

(3 marks)

- e. If this motorcycle is capable of decelerating at $1.1g$, at what height must its centre of gravity be?

(4 marks)

Question 2 - (Steady State Cornering)

- a. With the aid of diagrams, derive an equation for the 'ideal' lean angle of a motorcycle in steady state cornering.
(4 marks)
- b. Derive an equation, which expands on the concept of the ideal lean angle to include the motorcycles average tyre thickness into its required lean angle to give a more accurate representation of the required lean angle.
(6 marks)

Consider a motorcycle about to negotiate a right turn...

Explain in detail the process by which the rider gets from steady state straight-line motion, to steady state cornering.

- c. For a motorcycle speed of 10 ms^{-1}
(3 marks)
- d. For a motorcycle speed of 50 ms^{-1}
(3 marks)
- e. During the process of cornering a motorcycle, it is expected that both wheel velocities will become different to both the GPS vehicle speed and to each other. Explain, giving your reasoning, which velocity is best suited to traction control, and also explain why these 3 velocities change and in what order of magnitude they will end up during steady state cornering.
(4 marks)

Question 3 – (Suspension Analysis)

- a. For a normal front suspension fork show that the 'wheel rate' and the 'spring rate' are related by the equation.

$$k = k_f \cos^2 \varepsilon$$

(5 marks)

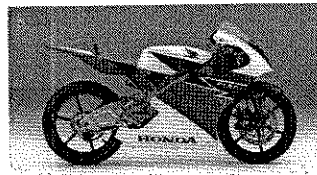
- b. For a rear suspension system, derive how the vertical 'wheel rate' and the shock 'spring rate' are related.

(5 marks)

- c. Assuming that during a track testing session, the front potentiometer measuring the fork movement develops a fault. Show in detail how you would go about trying to re-create a front suspension displacement channel from the changes in wheel speed of the bike.

(5 marks)

The Honda Moto3 motorcycle has a total mass of 84kg and a 58/42 forward weight distribution and a 63kg rider. It has a front unsprung mass of 6.4kg and a rear unsprung mass of 10.3kg. The rake angle is 22.36 degrees and the rear link ratio is an average of 2.32:1. Determine the following:



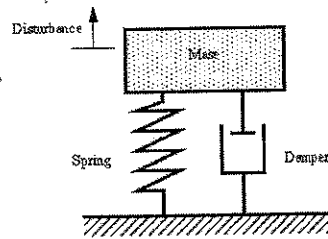
- d. Determine the vertical wheel stiffness's that are needed to satisfy a ride frequency of 2.0 Hz for the front and 2.2Hz for the rear.

(3 marks)

- e. Determine the critical damping that would be needed for these vertical stiffness's?

(2 marks)

Question 4 – (Dynamics and Damping)



The figure above shows a simple mass m attached to a spring of stiffness k and a viscous damper with a damping coefficient c . The displacement, x , is given by.

$$\frac{d^2x}{dt^2} + 2\zeta\omega \frac{dx}{dt} + \omega^2 x = 0$$

where $\omega = \sqrt{\frac{k}{m}}$ and the viscous damping factor $\zeta = \frac{c}{2m\omega}$

- Describe what you understand by the term 'Ride Frequency' (3 marks)
- What do you understand by the phrase 'Critical Damping'? (2 marks)
- Describe how the 'Damping Ratio' can be used to define damping characteristics. (3 marks)

- Show that the differential equation shown above, is exactly equivalent to the normal form used in motorcycle dynamics ' $ma + cv + kx = 0$ ', and solve this differential equation for $\zeta > 1$ where:

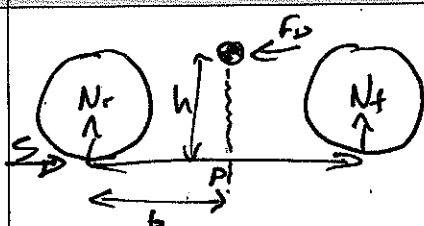
$$x = Ae^{\gamma_1 t} + Be^{\gamma_2 t}$$

(12 marks)



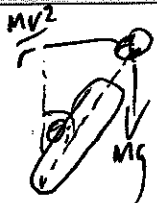
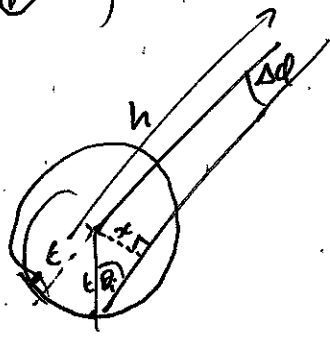
EXAMINATION SOLUTIONS : SESSION 2011/12

Programme:	BEng Motorcycle Engineering	Year:	Level 5
Module/Unit:	Motorcycle Dynamics		
Author:	Dr Owen Williams	Page:	1 of 4

Question No	Question	Marks
	 $N_r + N_f = mg$, moments about C of G $N_f(p-b) + Sh = N_r b$ $N_f(p-b) + Sh = mgb - N_f b$ $N_f p - N_f b + Sh = mgb - N_f b$ $N_f = mg \frac{b}{p} - S \frac{h}{p}$ Similarly $N_r = mg \left(\frac{p-b}{p} \right) + S \frac{h}{p}$ b Total force = $\frac{1}{2} \rho C_D A V^2 + 0.02mg \Rightarrow 425 \text{ N}$ c Including incline = $425 + mg \sin \theta \Rightarrow 1259 \text{ N}$ d Power = $\frac{1}{2} \rho C_D A V^3$ $V = 68 \text{ m/s}$ or 247 km/h e Sin braking = $ma + f_D$ $N_{r \text{ brake}} = mg \left(\frac{p-b}{p} \right) - S \frac{h}{p} = 0$ $mg \left(\frac{p-b}{p} \right) - (ma + f_D) \left(\frac{h}{p} \right) = 0$ $mg(p-b) - mah - F_D h = 0$ $h(ma + f_D) = mg(p-b)$ $h = \frac{mg(p-b)}{(ma + f_D)} = \frac{1154.14}{772.2} = 1.49 \text{ m}$	(8) (3) (2) (3) (4)

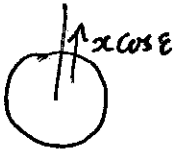
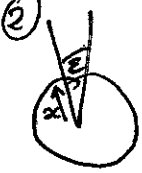
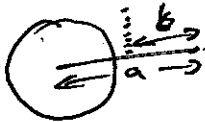
Continuation Sheet

Module/Unit:	Motorcycle Dynamics	Page:	2 of 4
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Question No	Question	Marks
a	 $\tan \theta_i = \frac{mv^2}{rmg} \Rightarrow \theta_i = \tan^{-1} \left(\frac{v^2}{rg} \right)$ $\text{or } \theta_i = \tan^{-1} \left(\frac{v^2 r}{g} \right)$	(4)
b	 $\sin \theta_i = \frac{x}{t} \quad \sin \Delta \theta = \frac{x}{h-t}$ $t \sin \theta_i = (h-t) \sin \Delta \theta$ $\frac{\sin \theta_i}{h-t} = \frac{\sin \Delta \theta}{t}$	(6) (3) (3)
c	Describe bicycle model of steering	(3)
d	Describe gyroscope model of steering	(3)
e	VGPS will be the true bike speed V_{front} will be higher due to reduced radius at Contact Patch V_{rear} will be highest for same reason (wide tyre) for traction control, VGPS is not suitable. GPS Dropout etc. for traction control V_{rear} referenced so V_{front} is preferred.	(4)
		20

Continuation Sheet

Module/Unit:	Motorcycle Dynamics	Page:	3 of 4
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Question No	Question	Marks
3a	①  ②  ① $N_f = K_f x \cos \epsilon$ ② $N_f \cos \epsilon = Kx$ Parallel N_f $K_f x \cos^2 \epsilon = Kx$ $K = K_f \cos^2 \epsilon$	5
b	 from $F = Kx$ $F_s = \frac{b}{a} F_w$ $x_s = \frac{a}{b} x_w$ $K_s = \frac{F_s}{x_s} = \frac{F_w \frac{b}{a}}{x_w \frac{a}{b}}$ so $K_s = \left(\frac{a}{b}\right)^2 K_w$	5
c	from V_{front}, differentiate to get V_{axel} (Combine V_f, V_r for better result) use $S = na$ to get force (s) use $N_f = mg \frac{b}{p} - S \frac{h}{p}$ to get N_f $F = Kx$ to get x vertically use x over $\cos \epsilon$ to get x at fork.	5
d	use $f = \frac{1}{2\pi} \sqrt{\frac{K}{m}}$ $K = (2\pi f)^2 m$ Remember % Spring - unsprung so $K_f = (4\pi)^2 \times (78.8)$ $K_r = (4.4\pi)^2 \times 51.4$ $K_f = 78.8 \text{ kg}$ $= 1245 \text{ N/mm}$ $= 9.8 \text{ N/mm}$ $K_r = 51.4 \text{ kg}$	3
e	Critical damp-$\gamma = 2\sqrt{K_m}$ $C_f = 2 \times \sqrt{K_m}$ $C_r = 2\sqrt{K_m}$ $C_f = 1481 \text{ Ns/m}$ $C_r = 1419 \text{ Ns/m}$	2
		20

Continuation Sheet

Module/Unit:	Motorcycle Dynamics	Page:	4 of 4
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Question No	Question	Marks
4a	Ride frequency $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ or $\omega = \sqrt{\frac{k}{m}}$ and is the undamped natural frequency of the body in mode. higher f = stiffer	(3)
b	Critical damping is when the spring in a system is damped such that it returns to its rest position without overshoot in the shortest possible time.	(2)
c	Damping ratio defines system behaviour $\zeta > 1$ = overdamped $\zeta < 1$ = underdamped $\zeta = 1 \Rightarrow$ Critical damping	(3)
d	$\frac{d^2 x}{dt^2} + 2\zeta\omega \frac{dx}{dt} + \omega^2 x = 0$ $\omega = \sqrt{\frac{k}{m}} \quad \zeta = \frac{c}{2\sqrt{km}}$ $\frac{d^2 x}{dt^2} + \frac{2c\omega dx}{2\sqrt{km}dt} + \frac{kx}{m} = 0 \Rightarrow \frac{d^2 x}{dt^2} + \frac{c\sqrt{k/m} dx}{\sqrt{km}dt} + \frac{kx}{m} = 0$ $\Rightarrow \frac{d^2 x}{dt^2} + \frac{c dx}{m dt} + \frac{k}{m} x = 0$ To Solve use $r^2 + 2\zeta\omega r + \omega^2 = 0$ quadratic formula $\Rightarrow r = -\zeta\omega \pm \omega\sqrt{\zeta^2 - 1}$ So $r_1 = \omega(-\zeta + \sqrt{\zeta^2 - 1})$ $r_2 = -\omega(\zeta + \sqrt{\zeta^2 - 1})$ So $x = Ae^{\omega(-\zeta + \sqrt{\zeta^2 - 1})t} + Be^{-\omega(\zeta + \sqrt{\zeta^2 - 1})t}$ Not knowing initial conditions, cannot find A, B so Complete.	(6)
		20